1 $f(x) = 2x - 3$ and $g(x) = x^2$

Show that $f^{-1}(55) = fg(4)$

[4 marks]

let $f(x) = y$

$y = 2x - 3$

$y + 3 = 2x$ (1)

$x = \frac{y+3}{2}$

$f^{-1}(x) = \frac{x+3}{2}$ (1)

$f^{-1}(55) = \frac{55+3}{2}$

$= \frac{58}{2}$

$= 29$ (1)

$fg(x) = 2(x^2) - 3$

$= 2x^2 - 3$ (1)

$fg(4) = 2(4)^2 - 3$

$= 2(16) - 3$

$= 32 - 3$

$= 29$

2 (a) $g(x) = 2x$ and $h(x) = \dfrac{x-1}{2}$

$hg(x) = \dfrac{2x-1}{2}$

Circle the expression for $hg(x)$

[1 mark]

$\dfrac{2x^2 - x}{2}$ $\quad$ $\dfrac{2x-1}{2}$ $\quad$ $x^2 - x$ $\quad$ $x - 1$

1

3 $f(x) = 2x + 5$

Show that $3f(x) - 12f^{-1}(x)$ simplifies to an integer.

[4 marks]

let $f(x) = y$

$y = 2x + 5$

$y - 5 = 2x$ (1)

$x = \frac{y-5}{2}$

$f^{-1}(x) = \frac{x-5}{2}$ (1)

$\therefore 3(2x+5) - 12\left(\frac{x-5}{2}\right)$ (1)

$= 6x + 15 - 6x + 30$

$= 45$ (1)

4 $f(x) = 3x$ and $g(x) = x^2$

Circle the expression for $fg(x)$

[1 mark]

$3x^2$ $9x^2$ $3x^3$ $9x^4$

1

5 $f(x) = x^2 + 6x$

$g(x) = 2x + 4$

5 (a) Show that $fg(x) = 4x^2 + 28x + 40$ **[3 marks]**

$fg(x) = (2x+4)^2 + 6(2x+4)$ (1)

$= 4x^2 + 16x + 16 + 12x + 24$ (1)

$= 4x^2 + 28x + 40$ (shown)

(1)

5 (b) Solve $fg(x) = -5$ **[3 marks]**

$4x^2 + 28x + 40 = -5$

$4x^2 + 28x + 45 = 0$ (1)

$x = \dfrac{-28 \pm \sqrt{28^2 - 4(4)(45)}}{2(4)}$ (1)

$= \dfrac{-28 \pm \sqrt{64}}{8}$

$= \dfrac{-28 \pm 8}{8}$ $= \dfrac{-20}{8}$ or $-\dfrac{36}{8}$

$= -2.5$ or -4.5

Answer -2.5 and -4.5 (1)

6 $f(x) = \frac{3x+9}{5}$ and $g(x) = 6x - 1$

6 (a) Show that gf(2) is an integer.

[2 marks]

$gf(x) = \frac{6(3x+9)}{5} - 1$ (1)

$= \frac{18x+54}{5} - 1$

$gf(2) = \frac{18(2)+54}{5} - 1$

$= \frac{36+54}{5} - 1$

$= 18 - 1 = 17$ (1)

6 (b) Show that $f^{-1}(8)$ is **not** an integer.

[2 marks]

let $f(x) = \frac{3x+9}{5}$

$y = \frac{3x+9}{5}$

$5y = 3x + 9$

$5y - 9 = 3x$

$x = \frac{5y-9}{3}$

$f^{-1}(x) = \frac{5x-9}{3}$ (1) $= \frac{5(8)-9}{3} = \frac{31}{3} = 10.3$ (1)

7 $f(x) = x - 3$ $g(x) = 4x - 7$

7 (a) Work out the value of fg(6)

[2 marks]

$fg(x) = (4x - 7) - 3$

$= 4x - 10$ ✓ (1)

$fg(6) = 4(6) - 10 = 14$ ✓ (1)

Answer 14

7 (b) Solve $(f(x))^2 = g(x)$

[4 marks]

$(x-3)^2 = 4x - 7$ ✓ (1)

$x^2 - 6x + 9 = 4x - 7$

$x^2 - 10x + 16 = 0$ ✓ (1)

$(x-8)(x-2) = 0$ ✓ (1)

$x = 8, x = 2$

Answer $x = 8$ and $x = 2$ ✓ (1)